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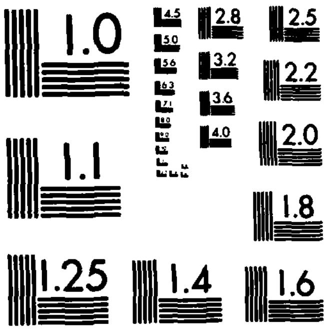
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This study is concerned with a high range resolution radar system for measuring the crab angle of landing aircraft. Previous attempts to measure the crab angle by means of one dimensional cross correlation techniques had proved to be unsuccessful, and the data from flight tests was examined to determine whether accurate estimates could be obtained by using two dimensional cross correlation techniques and centroids. Hundreds

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20. ABSTRACT (Continued)

of different types of estimators were examined, each one corresponding to different possibilities for data smoothing, data weighting, thresholding, and outlier removal; however, none of these methods produced satisfactory results. The probable cause for the poor performance was determined to be a lack of similarity in the returns from the two radars used in the system. The dissimilarity between the two returns was apparently caused by range differentials, aircraft roll, as well as the crab angle itself.

CONTENTS

CRAB ANGLE ESTIMATION WITH TWO-DIMENSIONAL
CROSS CORRELATIONS AND CENTROIDS

1. INTRODUCTION

In the Spring of 1982 the author was assigned the task of determining whether accurate crab angle estimates could be obtained by applying a certain two-dimensional cross correlation technique to the data described in Ref. [1]. The scope of the effort, and the general nature of the results obtained are described in Section 2. Hundreds of different types of crab angle estimators were examined, and they were all found to yield results which were consistently bad for large crab angles (5^o or more), and at best marginal for smaller angles. The probable cause for this poor performance is discussed in Section 2 and documented in Section 3, viz., a dissimilarity between the returns of the two radars used in the system caused by range differentials, aircraft roll, and the crab angle itself. In Sections 3-5, we discuss attempts to improve accuracy by numerically adjusting the thresholds on one or both of the radars, by various data weighting and data smoothing schemes, and by schemes for outlier removal. A representative sample of the results are tabulated in Sections 4 and 5. Although the similarity between the the two returns was often enhanced by applying these methods, the performance of the resulting estimators was never satisfactory. The explicit formulas used in the calculations are given in an appendix. A summary and conclusions are given in Section 6.

2. NATURE OF THE RESULTS

The crab angle (yaw angle) measurement system discussed in this report is described in Ref. [1]. It consists of the two X-band radars recommended in Ref. [1], which are located on opposite sides of the runway. For each emitted radar pulse recordings are taken of range and amplitude, and the data for each trial run therefore consists of two 2-dimensional arrays, AMP_1 (N,R) and AMP_2 (N,R) where AMP_I (N,R) is the (unsigned) amplitude of the detection made on the N'th pulse at the R'th range by the I'th radar. The ranges R were measured in increments of 1 ft., and the radar prf was 1000 Hz. The useful data was found to be within the bounds $0 \leq N \leq 700$ and $60<R<160$, and each array therefore consists of 70,000 entries.

Our task was to cross correlate the two arrays, find the lagged value ΔN of N which corresponds to the peak correlation, and then use this value to estimate the crab angle θ according to the formula (cf. Ref. [1], p. 10)

$$\theta = \tan^{-1} \frac{V \Delta t}{D} , \tag{1}$$

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where

V = aircraft speed,
D = distance between the radars = 210 ft., and
Δt = $\Delta N/1000$ = time lag corresponding to ΔN.

Previous attempts had been made to measure Δt by means of a certain one-dimensional cross correlation technique (Ref. [1]); however, the results were unsatisfactory, and it was hoped that the two-dimensional cross correlation technique would meet the accuracy goal of $\pm 1^o$. This hope was not realized, as will be seen.

The basic idea motivating this method is that the aircraft is symmetric about its fore and aft axis so that the two data records AMP_1 and AMP_2 should be approximately the same except for the time delay Δt produced by the crab angle. (See Ref. [1].) However, the two data records were found to differ significantly; namely, one radar always had more detections than the other, the peak correlations between the two records were very small, and the two auto-correlation functions of the records AMP_1 and AMP_2 (each correlated with a copy of itself at lagged values of N and R) were different. This lack of similarity between the two data records was apparently caused by range differentials as well as the crab angle itself. It also appears that the aircraft rolled as it maneuvered to produce a crab under conditions of zero crosswind. The roll was on the order of 5^o, and could have contributed to the lack of symmetry between the two data records. These results are documented in the next section.

Attempts were made to enhance the similarity between the records AMP_1 and AMP_2 by numerically adjusting the thresholds of one or both of the radars, and by various data smoothing and weighting schemes. These modifications are described, and the results documented, in the following sections. The results were again disappointing.

The correlation of two 70,000-element arrays is a formidable task. The correlation function of two 2-dimensional arrays is itself a 2-dimensional array, and hundreds of values (each being the sum of 70,000 products) must be calculated for accurate peak location by means of lobe smoothing and beam splitting. Because of its numerical complexity, even if the cross correlation technique has been successful it would have been highly desirable to examine other techniques which are easier to implement. The centroid is a natural candidate, which besides being easy to compute, also has a certain attractive stability property; viz., it is not very sensitive to data elisions. There are various kinds of centroids, corresponding to the different possibilities for thresholding and data weighting mentioned above. In addition, various schemes of outlier removal were used to enhance the similarity between the records AMP_1 and AMP_2. The results obtained with centroids were no better and no worse than those obtained by the correlation techniques.

The various methods for data thresholding, smoothing, and weighting, etc., can be combined in a large number of ways to produce different types of crab angle estimators. In all, more than 300 types were examined and the results were never of acceptable performance.

3. PRELIMINARY ANALYSIS OF THE DATA

Our initial examination of the data was confined to four trial data runs (Files 3, 5, 6, and 7) for which the crab angles were small, moderate, and large, the cross-track angles were negligible, and for which there were no problems in the recording system which required special treatment to reduce the data to usable form. The "true" values of crab angle and cross track angle were obtained from the optical data described in Ref. [1]. The sign convention is chosen so that the crab angle is positive for "left" crab (aircraft pointing to Radar 2) and negative for "right" crab (aircraft pointing to Radar 1). These values are shown in Table 1. The column labelled "SUM" is the signed sum of the crab angle and cross track angle, which is the apparent crab angle that would be sensed by the radar system. None of the methods produced acceptable results for the large crab angle in File 5. If, however, the results were acceptable or even marginal for the other "initial" files 3, 6, and 7, they were then tested against three "additional" files, labelled 4, 9 and 10. The data from the additional three files is always separated from that of the initial four files by a horizontal line. As it turned out, these additional files only served to confirm the previous results.

TABLE 1

Values of Crab Angle and Cross Track

	Crab Angle	Cross Track	SUM
File 3	$-.25^\circ$	0°	$-.25^\circ$
File 5	+5.5	-0.2	+5.3
File 6	+1.1	0	+1.1
File 7	-1.4	0	-1.4
File 4	-2.0	- .6	-2.6
File 9	-4.6	-1.4	-6.0
File 10	+1.8	+ .5	+2.3

We now focus on the lack of similarity between the records AMP_1 and AMP_2. Table 2 shows the total number of detections ND(1) and ND(2) made by Radars 1 and 2, and the ratio ND(2)/ND(1). There are six sets of data, labelled A-F. The first set "A" is derived from the original untouched data, and shows that Radar 2 consistently made more detections than Radar 1. This disparity was probably caused by the fact that the aircraft was 30-40 ft. closer to Radar 2 than to Radar 1. The sets labelled B-F show the number of detections that remain after the data has

been "reduced" in various ways in an attempt to enhance the similarity between the two records. In the sets B-E the outliers have been removed by calculating the means μ and second moments σ of each of the records $AMP_I(N,R)$ in the time (or pulse number N) direction, and then rejecting the points (N,R) whose N-coordinate lies outside the two limits $L = \mu \pm k\sigma$, where k is some fixed parameter. In the results shown, k = 1 or 2. By a simple modification of a DO loop parameter, the reduced data set can then be re-cycled through the subroutine any number of times, and we show some of the results for up to 3 cycles. The best results were obtained by using two cycles with k = 1. However, this equalization of ND(1) and ND(2) is purchased at the cost of a considerable reduction in the number of data points, which adversely affects the performance of the algorithms. The data set F was obtained by numerically raising the threshold level of Radar 2. It turned out that the disparity was almost completely reversed at the lowest possible threshold setting, so that the goal of equalizing the number of detections could not be attained in this way. In addition, we also examined the results of adjusting the threshold levels of both radars; however, we found that the accuracy of the corresponding crab angle estimates was not improved, and these results are not shown. In the sequel we shall show the results of applying various cross correlation techniques to the two sets A and F, and of using various centroid type estimates on the six sets A-F.

TABLE 2

Numbers of Detections

	A			B			C		
	Original Data			Outliers Removed $L = \mu \pm \sigma$; 1 cycle			Outliers Removed $L = \mu \pm 2\sigma$; 1 cycle		
	ND(1)	ND(2)	RATIO	ND(1)	ND(2)	RATIO	ND(1)	ND(2)	RATIO
File 3	747	855	1.14	497	517	1.04	723	839	1.16
File 5	665	876	1.32	429	549	1.28	641	859	1.34
File 6	776	889	1.15	523	557	1.07	741	868	1.17
File 7	827	921	1.11	544	571	1.05	799	908	1.14

	D			E			F		
	Outliers Removed $L = \mu \pm \sigma$; 2 cycles			Outliers Removed $L = \mu \pm \sigma$; 3 cycles			Threshold Adjust		
	ND(1)	ND(2)	RATIO	ND(1)	ND(2)	RATIO	ND(1)	ND(2)	RATIO
File 3	295	294	1.00	169	160	0.95	747	626	0.84
File 5	276	312	1.13	165	184	1.12	665	681	1.02
File 6	313	324	1.04	183	180	0.98	776	667	0.86
File 7	324	328	1.01	179	186	1.04	827	687	0.83

Another indicator of the dissimilarity between the two records is provided by the dissimilarity between their autocorrelation functions, each of which is obtained by cross correlating a record with a copy of itself at lagged values of N and R. The autocorrelation functions of $AMP_1(N,R)$ and $AMP_2(N,R)$ are shown in Figures 1 and 2. The data is from File 5. The entries are the values of the correlation coefficients multiplied by a factor of 100, except for the first column which shows the lagged value ΔN of N. The lagged values of ΔR increases as one moves horizontally from left to right, but their values are not shown since they are not involved in the calculations of the Δt used in Eq. (1). The formulas used to calculate the correlation functions and centroids are given in the Appendix. The peak value of an autocorrelation is unity (by construction), and the next largest values are $p = .42$ in Figure 1 and $p = .20$ in Figure 2. This difference is significant.

The cross correlation function of AMP_1 with AMP_2 is shown in Figure 3. The peak value $p = .14$ occurs at $\Delta N = 50$, and is marked with an asterisk. The small size of this peak value is another indicator of the dissimilarity between the two records.

The column containing the peak value of p in Figure 3 displays a lobed structure which is very flat and noisy in appearance, which makes it unsuitable for accurate peak location. For this reason we applied various smoothing operations to the data, which are described in the next section.

4. RESULTS USING CROSS CORRELATION

Two types of data smoothing schemes were used in applying the cross correlation technique. First, the (N,R)-space was divided into rectangular blocks consisting of n successive pulses and r contiguous range bins, and the amplitudes in each block were summed to form two new arrays which are then cross correlated. The advantage of this technique is that it produces a correlation function whose lobes are sharper and smoother than those obtained from the original data. The disadvantage is that it introduces a quantization error which increases with increasing n. Second, the correlation function itself was smoothed by applying a 5 point moving average along each column, (i.e., fixed range lag).

The results for $n = 5$ and $r = 3$ are shown in Table 3. The aircraft velocity was approximately 250 ft./sec, and referring to Eq. (1) with the indicated value of D, it follows that the quantization error for $n = 5$ is approximately 0.34^{o}. Other values of n and r sometimes produced a better lobed structure, but the corresponding angle estimates were never satisfactory. Three different values are tabulated, corresponding to three different methods for peak location:

(M). The location at which p is maximum.
(MAM). The location at which the 5 point moving average is maximum.
(BS). The result obtained by beam splitting the 5 point moving average by taking the average of the two 3 dB points.

For convenience, the "true" values of apparent crab angle are shown in parentheses; i.e., from the "sum" column of Table 2. Table 4 shows the

results when the same technique is applied to data set obtained by applying the threshold adjustment discussed above in constructing the data set "F" of Table 2. The results are exactly the same except at two places.

TABLE 3

Cross Correlation Estimates

	M	MAM	BS
File 3 (-0.25)	0°	+0.37	$+0.56^{\circ}$
File 5 (+5.3)	-1.48	-1.48	+2.22
File 6 (+1.1)	+0.68	+1.03	+1.03
File 7 (-1.4)	+0.35	+0.35	-0.35
File 4 (-2.6)	-0.10	-0.64	-0.96
File 9 (-6.0)	+8.17	+8.17	+8.46
File 10 (+2.3)	+2.28	+2.59	+1.66

TABLE 4

Estimates with Threshold Adjust

	M	MAM	BS
File 3	0°	$+0.37^{\circ}$	+0.56
File 5	-1.48	-1.48	+2.22
File 6	+0.68	+1.03	+1.03
File 7	+0.35	0	-0.35
File 4	-0.10	-0.64	-0.96
File 9	+8.17	+8.17	+8.46
File 10	+2.28	+2.59	+1.97

Among the first four "initial" files, the results are always bad for the large crab in File 5, but marginally acceptable results are contained in the BS column for Files 3, 6, and 7. The optical data from File 5 was examined very closely, and the bad results obtained from the "additional" Files 4 and 9 show that File 5 was not an anomaly. The result for File 9 is particularly bad because of the sign reversal.

Tables 5 and 6 show the results of applying the same calculations to "binary weighted" data, i.e., data for which $AMP_I(N,R)$ is replaced by 0 or 1 depending on whether a detection occurs at the indicated value of N and R. This technique produced an increase in the peak values of p, but did not improve the crab angle estimates. The peak values of p are shown in Table 7, for both the ordinary and binary weighted data. The simple maxima of the cross correlation functions are denoted by "M", and the maxima of the 5 point moving average by "MAM". The effects of threshold adjustment on the peak values of p were miniscule, and are not shown. The peak values of p for Files 4, 9, and 10 (with "ordinary" data) are .44, .43, and .40, respectively, and the observed tendency for the peak values of p to decrease as the magnitude of the crab angle increases is very slight, probably due to the complicating effects of range differentials and aircraft roll.

TABLE 5

Cross Correlation Estimates of Crab Angle with Binary Data

	M	MAM	BS
File 3 (-0.25)	+0.56°	+0.56°	+1.85°
File 5 (+5.3)	-1.85	-1.85	-0.37
File 6 (+1.1)	+0.85	+0.34	-0.51
File 7 (-1.4)	+0.35	+0.69	-0.17

TABLE 6

Cross Correlation Estimates of Crab Angle Data with Binary Data and Threshold Adjust

	M	MAM	BS
File 3	+0.74°	+0.74°	+1.48°
File 5	-1.85	-1.85	-0.56
File 6	+0.85	+0.34	-0.34
File 7	+0.35	+0.35	-0.17

TABLE 7

Peak Values of Correlation Coefficient p

	ORDINARY DATA		BINARY DATA	
	M	MAM	M	MAM
File 3	.44	.41	.51	.50
File 5	.36	.33	.44	.42
File 6	.48	.45	.57	.56
File 7	.52	.49	.60	.59

Figure 4 shows the autocorrelation function used in obtaining the results for File 5 in Table 3. Asterisks mark the locations of the peak and the two 3 dB points of the 5 point moving average. The maximum value in an adjoining column was very close to the peak, and is marked with a solid disk. Its location is closer to that corresponding to the actual crab angle. By reason of the quantizing of the (N,R)-space into 5x3 rectangular blocks, each unit increment of lag ΔN corresponds to a time delay Δt of 5 ms. Therefore the "true" peak location occurs at $\Delta N = 15$. Figure 5 shows the one-dimensional slice of the correlation function taken along the main column (containing the peak). The estimates for File 5 were very bad and Figure 5 shows how this happened: The correlation function for this particular file is non-symmetric and has multiple peaks.

Numerous other cross correlation estimators were examined besides those described above, viz., (1) different values for the parameters (n,r) in the (N,R)-space quantization, (2) different thresholds applied

to one or both radars, (3) logarithmic weighting (replacing $AMP_I(N,R)$ with its logarithm), (4) one dimensional correlation obtained by weighting each pulse by the number of detections or sum of all values (setting $r = \infty$ in (1)). These four techniques can be combined in numerous ways, but none of the dozens examined produced acceptable results.

5. CENTROIDS

The centroids (or means) μ_1 and μ_2 of the arrays AMP_1 and AMP_2 were calculated according to formulas given in the appendix, and the corresponding crab angle estimates were the obtained by setting $\Delta t = (\mu_2 - \mu_1)/1000$ in Eq. (1). Three types of weighting scheme were used in the calculation of centroids:

Type 1. Each pulse weighted by the number of detections

Type 2. Each pulse weighted by 0 or 1 depending on whether a detection occurred in that pulse.

Type 3. Each detection weighted by $AMP_I(N,R)$.

Type 3 is the more conventionally defined centroid. In addition to these three weighting schemes, there are six categories A-F which correspond to those in Table 2. We recall that A is obtained from the original untouched data, B-E from data with outliers removed, and F from data which has been threshold adjusted as discussed in Section 3. There are therefore 18 different kinds of centroids. The results are shown in Table 8. The results for the "additional" Files 4, 9, and 10 were only calculated for the categories A, B, C.

Again, other types of centroids were calculated by using logarithmic weights, by adjusting the thresholds on one or both radars, and by varying the parameters in the outlier removal scheme. But, as before, acceptable results were never achieved.

6. SUMMARY AND CONCLUSIONS

The system concept is based on the idea that the returns from the two radars should be similar except for the time lag produced by the crab; however, the two radar returns were found to be dissimilar. One radar always had more detections than the other, the two autocorrelation functions were different, and the peak values of the cross correlation were small. This dissimilarity between the two records was probably caused by range differentials, aircraft roll, and the crab angle itself.

In addition to the examination of cross correlation techniques, we also considered the use of estimates derived from the centroids, which are much easier to implement numerically.

Hundreds of different types of crab angle estimators were examined, each one corresponding to different possibilities for thresholding, data weighting, data smoothing, and outlier removal. None of these techniques

TABLE 8

Crab Angle Estimates Derived from Centroids

	A: Original Data			B: Outliers Removed $L = \mu \pm \sigma$; 1 cycle			C: Outliers Removed $L = \mu \pm 2\sigma$; 1 cycle		
	Type 1	Type 2	Type 3	Type 1	Type 2	Type 3	Type 1	Type 2	Type 3
File 3 (-0.25)	-0.01	+1.06	+0.84	-0.57	-0.52	-0.59	-0.45	+0.13	+0.80
File 5 (+5.3)	+2.08	+2.04	+2.59	+2.33	+2.33	+2.82	+2.15	+2.39	+2.95
File 6 (+1.1)	+0.58	+1.43	+2.36	+0.32	+0.31	+0.45	+0.32	+0.76	+2.37
File 7 (-1.4)	-0.19	+0.66	+0.16	-0.61	-0.59	-0.07	-0.48	-0.02	+0.12
File 4 (-2.6)	-1.30	-1.07	-1.18	-1.99	-1.99	-2.07	-1.27	-0.90	-0.87
File 9 (-6.0)	-2.22	-2.58	+2.58	-1.98	-2.01	-2.54	-1.78	-1.39	+2.75
File 10 (+2.3)	+3.42	+2.23	+4.51	+3.75	+3.74	+3.71	+4.15	+4.23	+4.83

	D: Outliers Removed $L = \mu \pm \sigma$; 2 cycles			E: Outliers Removed $L = \mu \pm \sigma$; 3 cycles			F: Threshold Adjust		
	Type 1	Type 2	Type 3	Type 1	Type 2	Type 3	Type 1	Type 2	Type 3
File 3 (-0.25)	-0.58	-0.56	-0.99	-0.57	-0.59	-0.75	-0.10	+0.65	+0.90
File 5 (+5.3)	+2.77	+2.81	+3.28	+2.88	+2.89	+3.08	+2.07	+1.75	+2.65
File 6 (+1.1)	+0.02	0	+0.23	+0.13	+0.14	+0.29	+0.41	+0.87	+2.42
File 7 (-1.4)	-0.39	-0.38	-0.21	+0.03	+0.03	+0.26	-0.16	+0.46	+0.21

produced acceptable results, and the performance at the two largest crab angles (5^{o} and 6^{o}) was always extremely poor. We therefore conclude that any further examination of this system would not be warranted.

7. REFERENCES

(1) F. Donald Queen and James J. Alter, Results of a Feasibility Study for Determining the Yaw Angle of a Landing Aircraft, NRL Report 8480, May 27, 1981.

-10	0	2	3	3	1	0	2	0	4	11	34	8	5	0	1	1	0	0	1
-9	0	2	1	5	1	0	1	0	5	14	26	10	8	0	0	1	1	1	1
-8	1	0	3	4	1	0	1	0	6	12	26	15	6	0	0	2	1	1	1
-7	0	1	3	4	1	0	1	0	5	14	29	13	7	1	0	1	1	1	1
-6	0	1	3	3	1	0	2	0	6	11	30	15	6	0	0	1	1	1	1
-5	1	1	3	3	1	1	1	0	6	7	43	9	6	0	0	0	0	1	2
-4	0	2	1	3	1	1	1	0	5	14	36	13	7	0	1	0	0	1	2
-3	1	1	2	3	1	0	1	0	6	16	36	16	6	0	0	1	0	1	2
-2	0	2	2	2	1	1	0	0	4	18	39	12	10	0	0	1	0	2	1
-1	1	1	2	2	1	0	0	0	7	13	42	21	4	0	0	1	0	2	1
0	1	1	2	1	0	0	0	0	0	0	100	0	0	0	0	0	0	1	2
1	0	2	1	2	0	1	0	0	4	21	42	13	7	0	0	0	1	2	2
2	1	1	1	2	0	1	0	0	10	12	39	18	4	0	0	1	1	2	2
3	1	1	2	1	0	1	0	0	6	16	36	16	6	0	1	0	1	3	2
4	1	1	2	1	0	0	1	0	7	13	36	14	5	0	1	1	1	3	1
5	0	0	2	1	0	0	0	0	6	9	43	7	6	0	1	1	1	3	3
6	0	1	1	1	1	1	0	0	6	15	30	11	6	0	2	0	1	3	3
7	1	0	1	1	1	1	0	1	7	13	29	14	5	0	1	0	1	4	3
8	0	1	1	1	1	2	0	0	6	15	26	12	6	0	1	0	1	4	3
9	1	0	1	1	1	1	0	0	8	10	26	14	5	0	1	0	1	5	1
10	0	0	1	0	0	1	1	0	5	8	34	11	4	0	2	0	1	3	3

Figure 1. Autocorrelation Function for AMP_1 (N,R) of File 5

-10	0	1	2	1	4	5	4	6	4	11	11	12	6	5	4	4	1	3	0
-9	1	1	1	3	5	6	2	4	9	10	14	9	6	4	6	4	2	2	1
-8	1	1	2	3	3	4	3	5	8	8	14	12	7	5	4	5	3	2	1
-7	0	1	1	2	3	5	2	5	6	12	11	11	10	3	6	3	2	0	2
-6	0	1	2	2	3	3	5	4	5	13	13	14	6	5	4	5	2	2	0
-5	0	0	1	2	3	3	6	5	7	10	16	10	7	6	5	4	2	2	1
-4	1	1	1	1	2	6	6	3	6	11	20	7	10	2	6	4	3	2	1
-3	1	1	0	1	3	6	3	6	6	11	12	12	8	5	5	2	3	2	1
-2	0	1	1	1	2	5	5	4	8	11	16	15	5	3	7	4	4	1	0
-1	1	0	1	2	4	1	4	7	8	15	20	14	6	6	2	5	3	2	1
0	0	0	0	2	6	3	4	1	0	0	100	0	0	1	4	3	6	2	0
1	1	0	1	2	3	5	2	6	6	14	20	15	8	7	4	1	4	2	1
2	1	0	0	1	4	4	7	3	5	15	16	11	8	4	5	5	2	1	1
3	0	0	1	2	3	2	5	5	8	12	12	11	6	6	3	6	3	1	0
4	0	1	1	2	3	4	6	2	10	7	20	11	6	3	6	6	2	1	1
5	1	0	1	2	2	4	5	6	7	10	16	10	7	5	6	3	3	2	1
6	0	0	0	2	2	5	4	5	6	14	13	13	5	4	5	3	3	2	2
7	1	0	2	0	2	3	6	3	10	11	11	12	6	5	2	5	3	2	1
8	0	0	1	2	3	5	4	5	7	12	14	8	8	5	3	4	3	3	2
9	0	1	1	2	2	4	6	4	6	9	14	10	9	4	2	6	5	3	1
10	1	1	0	3	1	4	4	5	6	12	11	11	4	6	4	5	4	1	2

Figure 2. Autocorrelation Function for AMP_2 (N,R) of File 5

25	1	2	4	7	7	4	5	4	6	3	4	4	3
26	2	2	3	8	7	4	6	1	7	5	7	1	3
27	1	3	3	7	8	3	5	3	6	6	6	2	3
28	2	2	4	6	7	4	3	6	4	9	4	3	3
29	1	3	4	7	8	3	3	6	7	6	5	3	3
30	2	3	5	6	9	4	4	5	9	2	6	3	4
31	1	4	3	9	7	6	3	3	10	4	6	3	4
32	1	3	5	8	8	5	2	5	11	5	4	3	3
33	2	4	4	8	10	4	2	4	5	7	7	4	2
34	1	3	5	7	10	3	5	3	8	9	4	3	3
35	2	3	5	8	7	6	3	5	11	4	5	4	4
36	1	4	6	7	6	6	3	4	9	6	5	3	2
37	3	3	6	7	7	7	3	4	8	8	4	4	4
38	1	4	6	6	7	4	4	4	6	8	5	3	4
39	1	3	7	6	8	4	4	4	7	5	8	4	2
40	1	3	6	7	6	6	4	5	7	3	8	5	2
41	2	2	7	5	7	7	3	3	9	5	8	3	3
42	2	3	5	6	7	5	4	3	10	4	6	3	3
43	1	3	5	6	6	5	4	6	7	7	7	4	2
44	1	2	7	6	7	3	4	5	9	4	7	4	2
45	1	3	6	6	5	4	4	6	12	3	6	3	2
46	2	3	6	6	5	3	7	4	9	4	8	3	2
47	1	2	7	7	6	4	3	3	11	6	5	3	2
48	2	3	6	7	7	3	4	5	7	6	5	3	4
49	2	3	7	7	7	2	4	6	7	6	4	3	3
50	2	4	7	5	6	3	5	4	14*	2	4	5	3
51	2	4	5	5	6	6	4	3	10	5	3	5	2
52	4	4	4	7	6	3	5	5	9	5	4	4	3
53	2	2	5	8	5	4	4	4	5	6	5	4	2
54	3	3	5	6	6	3	5	4	6	8	4	4	2
55	2	4	4	5	7	4	4	5	7	4	4	5	3
56	2	3	5	5	6	6	3	5	6	5	6	5	3
57	2	3	6	5	7	3	5	5	8	3	6	6	2
58	3	3	5	5	7	4	4	6	4	5	6	6	2
59	2	2	7	6	6	4	3	6	6	5	7	4	2
60	3	4	5	4	8	3	3	7	6	5	6	5	2
61	3	4	5	5	6	5	6	4	6	5	6	5	1
62	4	3	4	5	6	7	6	3	6	5	5	6	2
63	6	3	6	7	4	4	5	6	4	5	6	4	3
64	5	4	5	5	8	4	3	8	5	5	4	6	2
65	3	4	5	6	7	4	6	5	5	5	4	4	2
66	2	4	4	6	7	5	4	6	4	4	3	4	4
67	5	4	4	6	6	5	4	6	6	3	5	2	3
68	4	3	4	7	6	5	4	6	3	4	4	4	4
69	2	4	5	7	8	3	5	5	3	5	3	7	2
70	2	5	3	6	7	6	3	4	6	2	4	4	3
71	4	4	3	5	7	5	5	5	3	4	5	4	3
72	3	3	4	7	5	4	5	6	3	4	4	3	4
73	2	5	4	6	6	7	5	5	3	4	4	4	3
74	4	3	3	5	8	6	4	3	5	5	5	5	4
75	2	3	2	7	5	4	5	5	3	4	6	5	2

Figure 3. Cross Correlation of AMP_1 with AMP_2 for File 5

-17	1	4	21	20	10	9	4	5	6	3	1
-16	1	4	22	20	10	12	6	6	6	2	1
-15	1	7	20	20	11	13	6	6	6	3	1
-14	1	9	19	23	12	17	6	8	6	2	1
-13	1	13	18	27	12	18*	7	9	6	3	1
-12	1	12	15	29	11	19	8	11	6	2	2
-11	1	11	16	26	11	22	9	11	7	2	2
-10	1	13	17	27	10	26	11	11	6	3	2
-9	1	14	19	25	10	29	13	10	6	3	2
-8	1	13	20	24	9	32	12	9	5	3	2
-7	2	13	18	28	9	31	11	9	5	3	3
-6	2	11	14	30	10	32	10	9	5	3	3
-5	2	9	12	27	11	33	13	10	4	2	3
-4	2	7	11	28	10	36*	16	11	5	2	3
-3	3	6	12	25	12	34	17	11	5	2	3
-2	3	6	13	20	12	32	20	10	6	2	3
-1	3	7	14	17	11	31	24	10	7	2	3
0	2	8	13	15	15	28	25	12	9	2	3
1	2	10	12	14	17	27	27	14	9	2	2
2	2	12	11	15	18	26	26	15	8	2	2
3	3	9	10	16	19	26	25	14	8	2	2
4	4	7	9	14	20	27	26	14	9	1	2
5	5	6	7	13	19	27	28	16	8	1	2
6	5	7	7	11	20	28	32	18	8	2	2
7	5	8	7	11	23	30	33	19	7	2	2
8	6	10	8	13	23	28	33	20	6	1	2
9	6	12	8	16	22	24	34	21	5	2	2
10	7	10	6	19	23	24	34	19	5	2	2
11	6	8	6	19	23	24	29	22	6	2	3
12	6	6	6	19	23	24	29	25	8	2	3
13	7	5	5	18	22	26	27	23	9	3	3
14	6	5	6	18	21	27	23	21	9	3	3
15	7	5	6	18	19	26	23	24	11	2	3
16	6	4	6	16	18	24	22	28	10	3	3
17	6	5	5	15	17	24	20	29	10	3	4
18	6	4	6	18	16	25	20	25	11	3	3
19	6	4	5	16	16	26	21	20	13	3	3
20	5	4	5	14	15	26	20	16	13	4	3
21	4	4	5	12	13	24	21	15	15	5	4
22	5	3	6	11	14	22	21	14	16	5	4
23	4	3	6	11	14	22	20	17	17	5	5
24	4	3	6	8	12	20	18	22	17	6	7
25	5	3	7	8	12	16*	18	19	17	9	7
26	5	3	6	7	10	17	16	17	18	10	7
27	5	3	7	6	10	14	16	19	19	11	7
28	4	2	6	7	8	13	16	17	21	13	8
29	4	3	6	6	10	12	16	12	23	14	8

Figure 4. Cross Correlation Function for the Quantized Data from File 5

$n = 5 \quad r = 3$

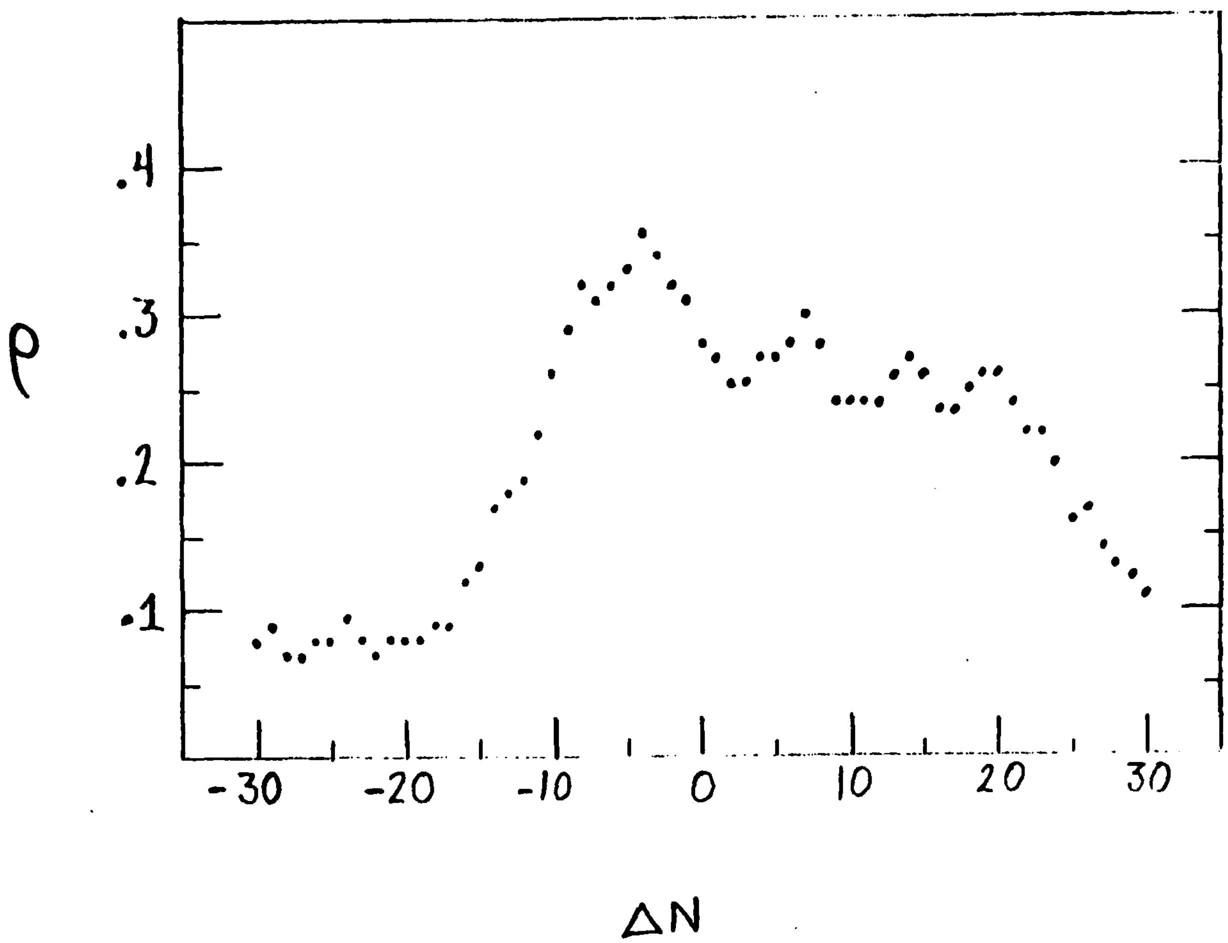

Figure 5. Main Column of the Cross Correlation Function Shown in Figure 4

APPENDIX

The cross correlation coefficient $p(\Delta N, \Delta R)$ was computed according to

$$p(\Delta N,\Delta R)=\frac{\sum_{N=N1}^{N2}\sum_{R=R1}^{R2}\left[AMP_1(N,R)\cdot AMP_2(N+\Delta N,\ R+\Delta R)\right]}{\left\{\sum_{N=N1}^{N2}\sum_{R=R1}^{R2}\left[AMP_1(N,R)\right]^2\right\}^{1/2}\cdot\left\{\sum_{N=N1}^{N2}\sum_{R=R1}^{R2}\left[AMP_2(N+\Delta N,\ R+\Delta R)\right]^2\right\}^{1/2}}. \quad \text{(A-1)}$$

The limits of summation N1, N2, R1 and R2 vary with ΔN and ΔR, and are chosen so that, in effect, the data is zeroed where the two records do not overlap. In the calculation of p with binary weighted data, we merely set AMP_1 and AMP_2 equal to 0 or 1 depending on whether a detection occurs at the indicated value of the arguments. It is a common practice to subtract the means from the amplitudes in this calculation. However, this procedure does not affect the location of the peak.

The two centroids μ_1 and μ_2 of Type 1 were calculated according to

$$\mu_I = \frac{\sum_N N \cdot v_I(N)}{\sum_N v_I(N)} , \quad \text{(A-2)}$$

where $v_I(N)$ is the number of detections at the N'th pulse of the I'th radar. The centroids of Type 2 are given by

$$\mu_I = \frac{\sum_N N \cdot \phi_I(N)}{\sum_N \phi_I(N)} , \quad \text{(A-3)}$$

where $\phi_I(N)$ is 0 or 1 depending on whether a detection occurs at the N'th pulse. The more conventionally defined centroids of Type 3 are given by

$$\mu_I = \frac{\sum_N \sum_R N \cdot AMP_I(N,R)}{\sum_N \sum_R AMP_I(N,R)} . \quad \text{(A-4)}$$

Note that the Type 1 centroids are equivalent to Type 3 centroids with binary weighted data.

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